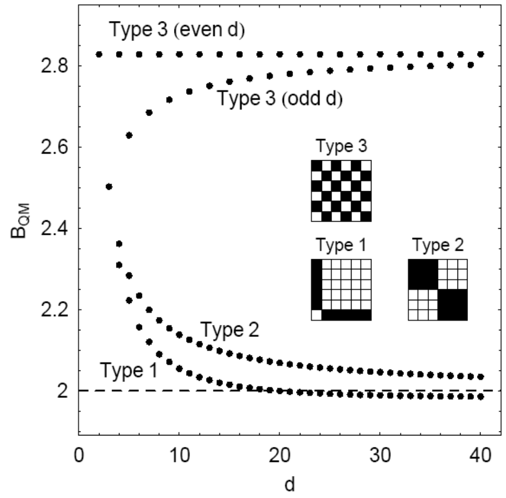
Oxford contribution

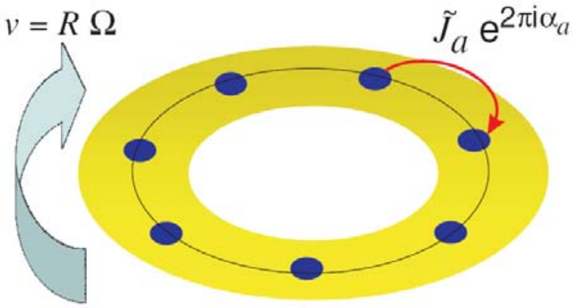
- Dieter Jaksch
- (University of Oxford, UK)

OLAQUI Topics

- Bell inequalities for high-dimensional quantum systems
  - ➡ Optimized Bell inequalities (WP 4)
  - ➡ Bell inequalities for squeezed states (WP 4)
  - ➡ Bell inequalities for imperfect detectors (WP 4)



- Direct quantum simulation with multi component degenerate gases in optical lattices
  - ➡ Polaron physics in optical lattices (WP 6)
  - ➡ Self trapping and stability (WP 6)
  - ➡ Simulation of artificial magnetic fields (WP 6)



Bell inequalities - Motivation

SW Lee and D Jaksch

- Bell inequalities are a very powerful means for investigating the connection between non-locality and entanglement in two-dimensional two-partite systems (e.g. CHSH)
- Can they be extended to optical lattice setups which are d-dimensional and N-partite?
- Some proposals for N=2 but none with all desired properties

	Maximal violation by MES	Tight
CHSH (2-dim.)	O	O
CGLMP (*d*-dim.)	X	O
SLK (*d*-dim.)	O	X
Optimal (*d*-dim.)	O	O
Continuous variable	O	O

Generalized Bell inequalities for *d*-dim

- Generalized Bell function in *d*-dimensions

$$\mathcal{B} = \sum_{a,b} \sum_{n_a, n_b = 0}^{d-1} f_{ab}(n_a, n_b) E_{ab}(n_a, n_b),$$

→ Correlation function representation

$$= \sum_{a,b} \sum_{k,l=0}^{d-1} \epsilon_{ab}(k,l) P(A_a = k, B_b = l),$$

→ Joint probability representation

where $E_{ab}(n_a, n_b) = \int d\lambda \, \rho(\lambda) \; (A_a(\lambda))^{n_a} \, (B_b^*(\lambda))^{n_b} = \sum_{k,l=0}^{d-1} \omega^{n_a k - n_b l} P(A_a = k, B_b = l).$

- This representation includes known types of BIs like CHSH, CGLMP, SLK
- Coefficients of the two presentations are related by a Fourier transform

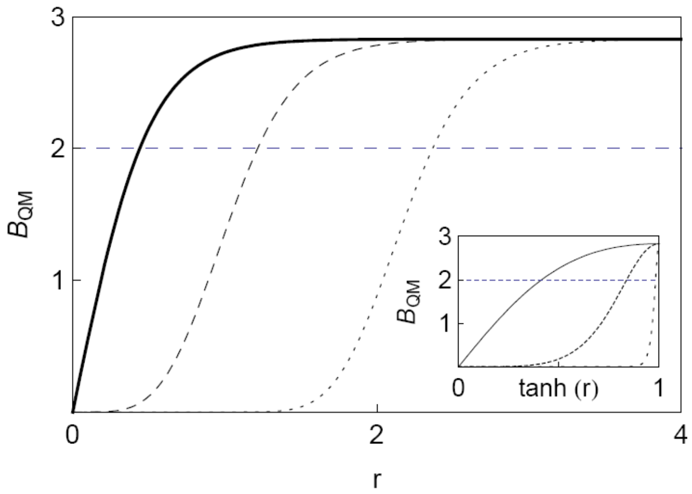
$$\epsilon_{ab}(k,l) = \sum_{n_a, n_b = 0}^{d-1} f_{ab}(n_a, n_b) \omega^{n_a k - n_b l}$$

Bell inequalities for squeezed states

- We use the optimal Bell inequality to study non-locality in two-mode squeezed states with squeezing parameter *r*

$$|\psi_s\rangle = \frac{\mathrm{sech} r}{\sqrt{1-\tanh^{2s+2} r}} \sum_{n=0}^{s} \tanh^n r |n, n\rangle$$

- Violation as a function of squeezing *r*



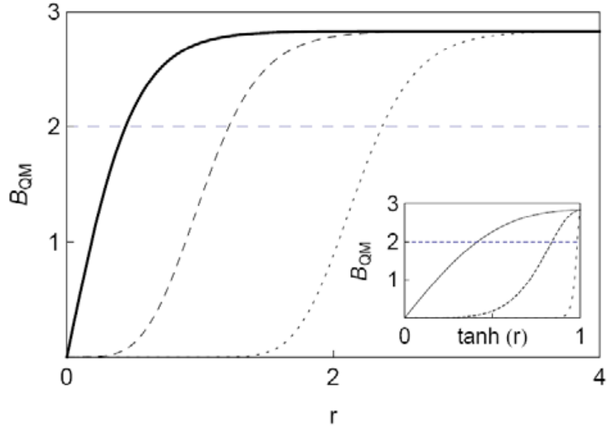
- OLAQUI spin squeezing experiments in Mainz

Optimal Bell inequality

- Deriving new types is equivalent to choosing coefficients of the general structure, e.g.

$$\epsilon_{11} = \zeta(R_1)\zeta(S_1), \quad \epsilon_{12} = \zeta(R_1)\zeta(S_2),$$
$$\epsilon_{21} = \zeta(R_2)\zeta(S_1), \quad \epsilon_{22} = -\zeta(R_2)\zeta(S_2),$$

$$\zeta(S) \begin{cases} 1 \text{ (inside } S) \\ -1 \text{ (outside } S) \end{cases}$$



- We choose $R_1=R_2=S_1=S_2=\{0; 2; 4; \ldots\}$ and find that the resulting Bell inequality is
  - ➡ (i) tight, and
  - ➡ (ii) maximally violated by maximally entangled states

$$\mathcal{B}_{QM} = \sum_{a,b=1}^{2} \sum_{k,l=0}^{d-1} \frac{\epsilon_{ab}(k,l)}{2d^3 \sin\left[\frac{\pi}{d}(k+l+\alpha_a+\beta_b)\right]}$$

OLAQUI spin squeezing experiments in Mainz

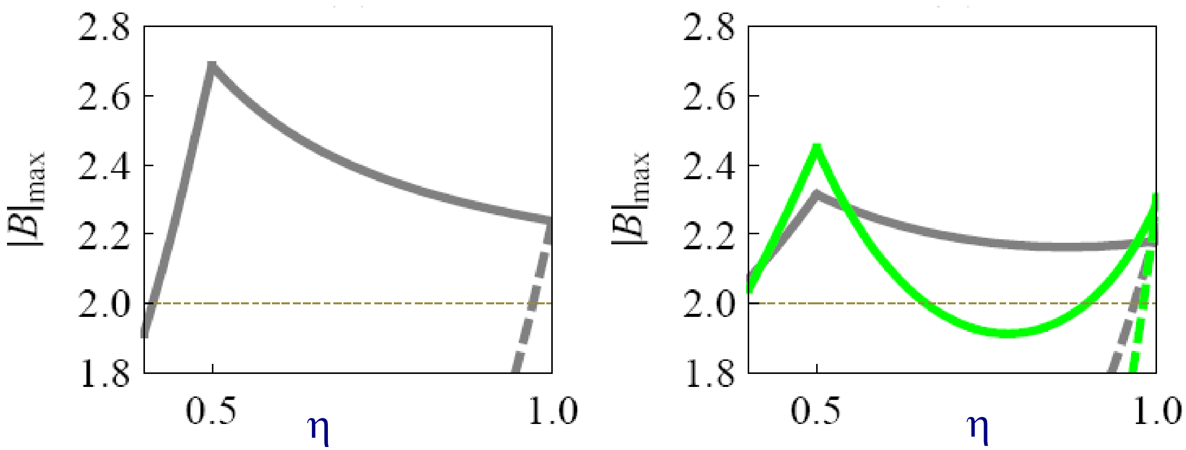
Continuous variable systems

- In the limit d → 1 we find that the EPR state (as the limit of a two mode squeezed state) maximally violates the optimal Bell inequality with measurement of quantum phase correlations

$$\hat{E}_{ab} = \sum_{k_a, l_b = 0}^{d-1} (-1)^{k_a + l_b} |\theta_{k_a}\rangle\langle\theta_{k_a}| \otimes |\theta_{l_b}\rangle\langle\theta_{l_b}|$$

- For imperfect detectors our method enables direct detection of non-locality in continuous variable systems by measuring quasi-probability distributions

$$W(\alpha; s) = -\frac{2}{\pi s} \int d^2\beta \; W(\beta) \exp\left(\frac{2|\alpha - \beta|^2}{s}\right)$$



Workpackage 4:
Generation and Characterization of multi-partite entangled states (**D9**)

Lattice immersed in a BEC - Motivation

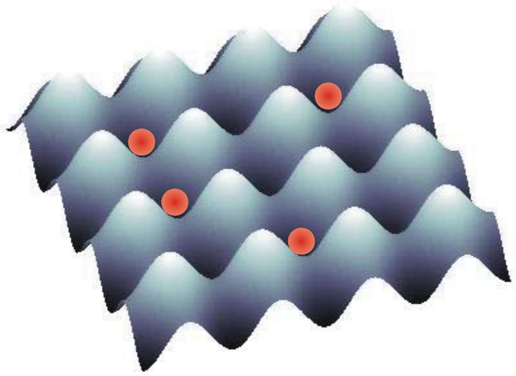
M Bruderer, A Klein, SR Clark and D Jaksch

- A BEC with interaction strength *g* exhibits phonons, which are well understood.
- Coupling κ between BEC phonons and lattice atoms is controllable by Feshbach resonances.

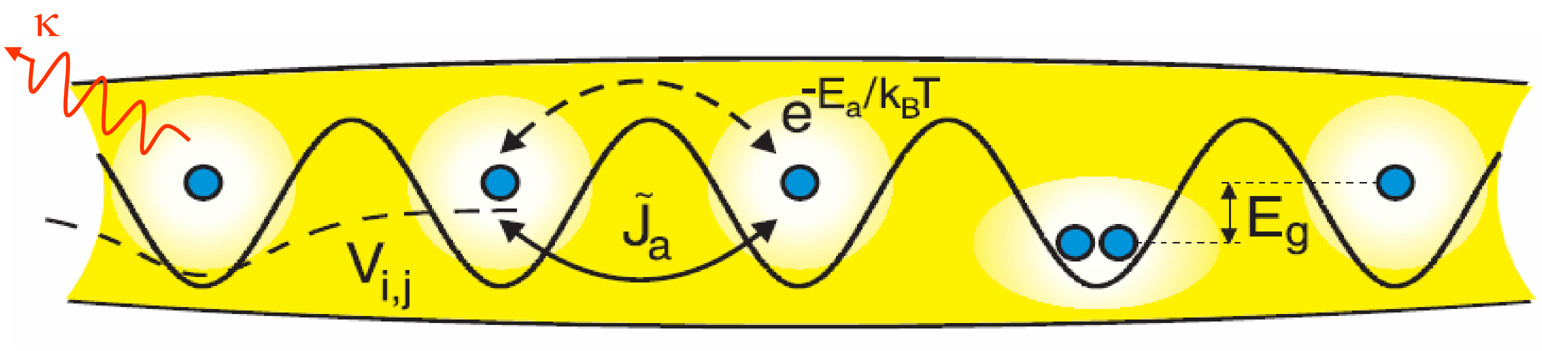
  ⇨ Introduce phonons into an optical lattice

- Study their influence on the dynamics of lattice atoms
- Phonon properties can be manipulated by the trapping of the BEC.

  ⇨ Simulate dynamics of condensed matter models (WP6)



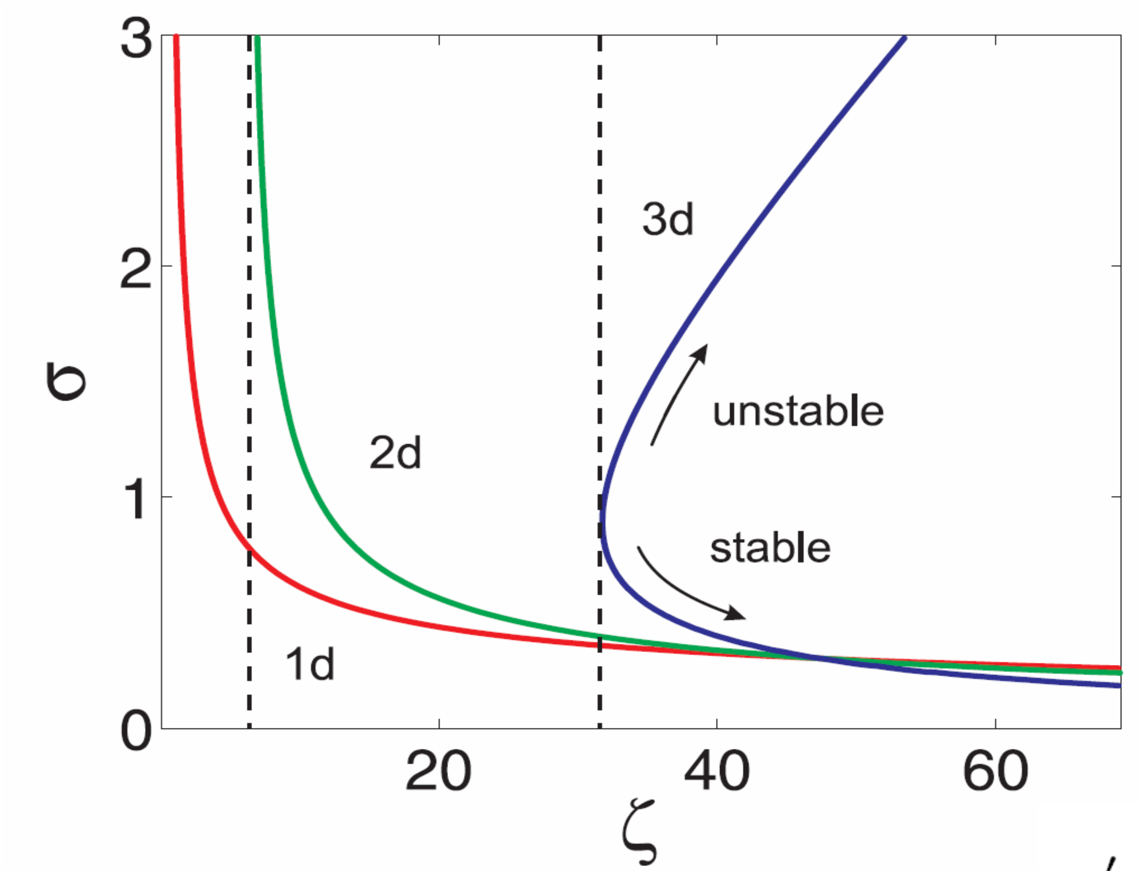
Self trapping



- Self trapping may alter the picture if it affects the lattice atom wave functions
  - ⇨ For repulsive interactions $\kappa > 0$ we find self trapping to have negligible effects
    - 3D and 2D self trapping only occurs above a certain interaction threshold
    - 1D self trapping will always occur
    - Self trapping will not compress the atomic wave function far beyond ξ
  - ⇨ For attractive interactions $\kappa < 0$ the solutions may be unstable
    - 3D: finite width solutions with $\kappa < 0$ are unstable
    - 2D: finite width solutions with $\kappa < \kappa_2$ are unstable
    - 1D: finite width solutions are stable

Self-trapping - perturbation theory

- We consider a single self-trapped impurity atom immersed in a BEC. For interaction strength ζ the width of the impurity wave function is σ given by



Self trapping for

$$\zeta > 2\pi \text{ in 2d}$$

$$\zeta \gtrsim 31.7 \text{ in 3d}$$

$\zeta = (\kappa/g)^2 \, m_a/m_b \, \xi^d \, n_0$

$$\varepsilon'\chi = -\frac{1}{2}\nabla^2\chi - \zeta|\chi|^2\chi$$

Variational approach for $\kappa<0$

- We study the total energy $E_{BEC} + E_{kin} + E_{int}$
- We assume a deformation of the BEC of the form

$$\delta\psi_\sigma(\mathbf{x}) = \frac{a}{\sigma^{\delta/2}} \prod_{j=1}^{d} \exp(-x_j^2/b\sigma^2)$$

- And use a Gaussian trial wave function for the impurity

$$\chi_\sigma(\mathbf{x}) = \left(\pi\sigma^2\right)^{-d/4} \prod_{j=1}^{d} \exp(-x_j^2/2\sigma^2)$$

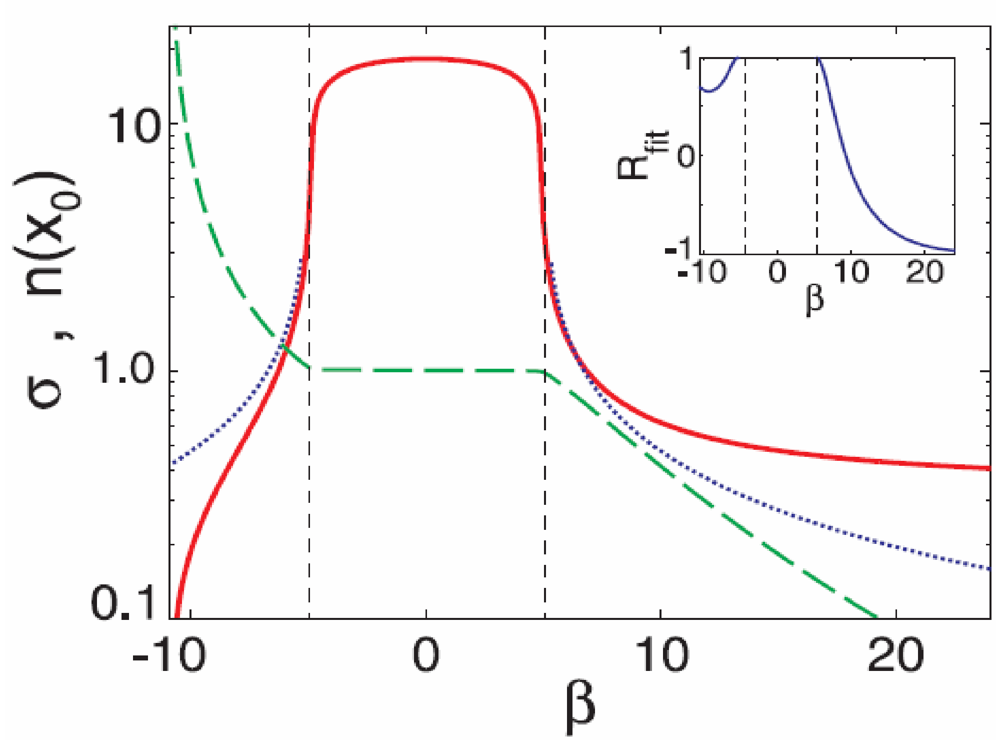
- In 3D: The total energy is bounded from below for σ ! 0. The total energy is dominated by the interactions for $2 < \delta < 3$. The BEC deformation tends to zero

$$\int \mathrm{d}\mathbf{x} |\delta\psi_\sigma(\mathbf{x})|^2 \;\rightarrow\; 0$$

- In 2D: For $\delta = 2$ all contributions to the energy scale like σ^{-2}. Whether an instability occurs depends on the system parameters. The BEC deformation is finite.
- In 1D: The energy is never dominated by the interaction energy.

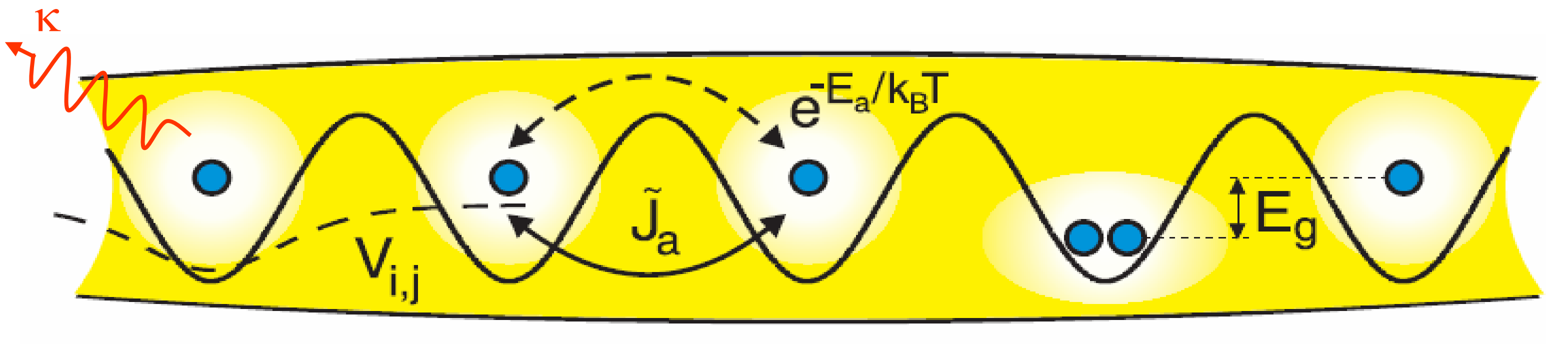
Self-trapping beyond perturbation theory

- Two spatial dimensions



Red curve: width of the impurity wave function σ
Long dashed green curve: density of the BEC at the impurity position
Blue dotted curve: Weak coupling approximation

Coherent atom hopping



- At low BEC temperatures the main effect of polaron formation is a reduction of the coherent hopping rate and changes to the interaction strength

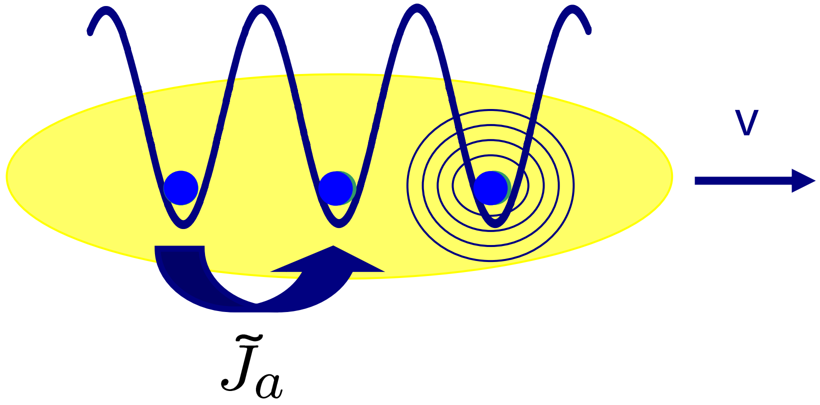
$$\tilde{J}_a \approx J\,\mathrm{e}^{-C\kappa^2 T}$$

- Importantly dephasing and dissipation can be kept very low for small BEC temperatures and do not significantly affect the optical lattice dynamics.

- Transport through optical lattices: OLAQUI transport experiments in Pisa
- Cooling and state engineering via lattice immersions: OLAQUI theory in IBK

Moving BEC

- Optical lattice in a moving BEC at very low temperature



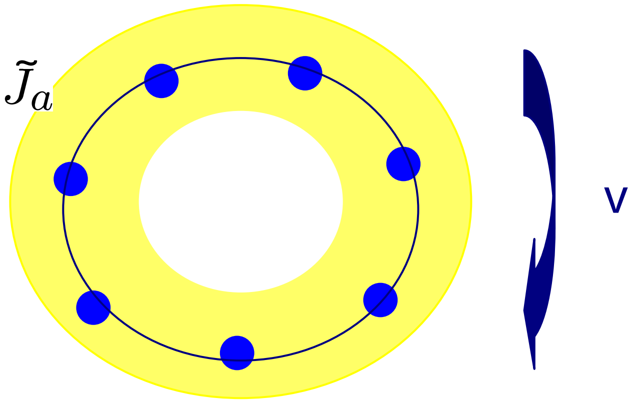
$$q \rightarrow q - mv$$

$$\tilde{J}_a \rightarrow \tilde{J}_a \, \mathrm{e}^{\mathrm{i}2\pi\alpha}$$

$$\alpha \propto v$$

⇨ In lowest order for v ¿ c this leads to a phase α proportional to v

- Rotating optical lattice immersions



Rotating BEC
Rotating lattice

→ No need to balance centrifugal terms

Moving background BECs

- We derive a Holstein-Hubbard model and find that the hopping rate is modified by correlations between BEC deformations at neighbouring lattice sites.
- These are described by unitary Glauber displacement operators of the phonon clouds are given by

$$\langle\langle \hat{X}_i^\dagger \hat{X}_j \rangle\rangle = \exp\left(-\frac{1}{2}\sum_q{}' |M_{i,q} - M_{j,q}|^2 (2N_q(T)+1)\right)$$
$$\times \exp\left(\frac{1}{2}\sum_q{}' M_{i,q}M_{j,q}^* - M_{i,q}^* M_{j,q}\right) ,$$

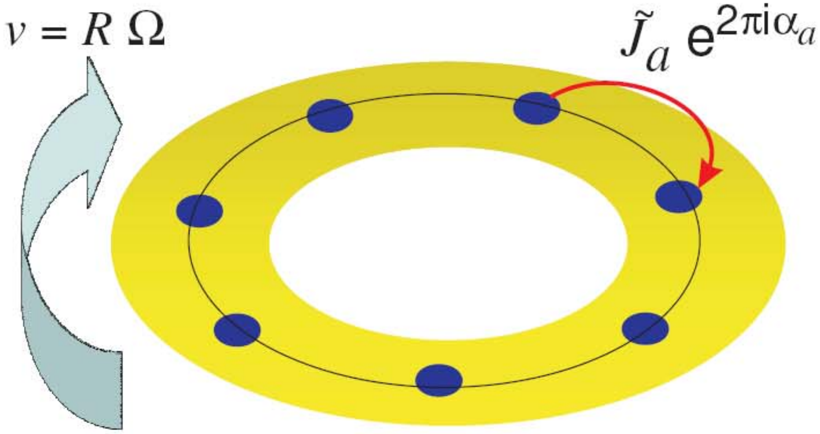
- The hopping term in the Bose-Hubbard Hamiltonian becomes

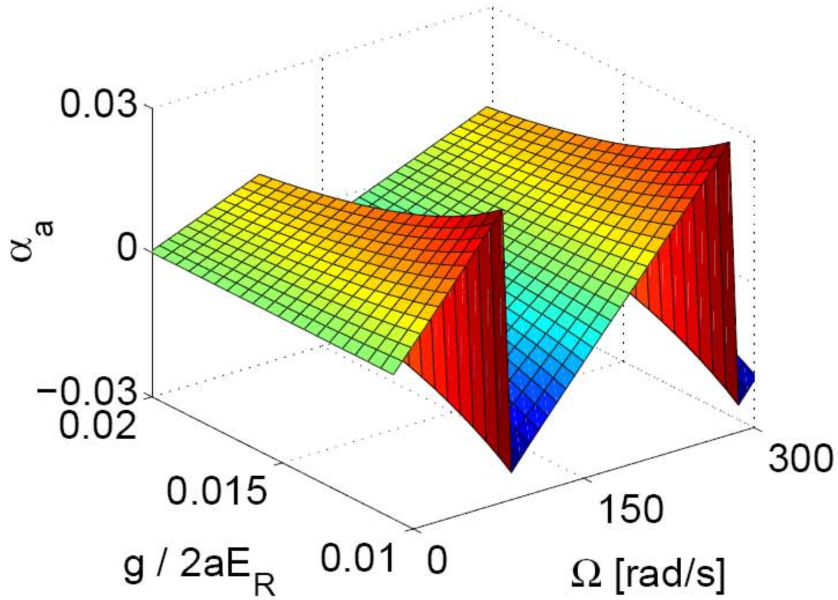
$$-\tilde{J}_a \sum_{\langle i,j \rangle} \exp(2\pi \mathrm{i} \alpha_{i,j}) \hat{a}_i^\dagger \hat{a}_j$$

which contains a phase of

$$\alpha_{i,j} = 1/(4\pi \mathrm{i}) \sum_q{}' (M_{i,q}M_{j,q}^* - M_{i,q}^* M_{j,q})$$

Lattice on a ring





- For a Wannier function of width σ we thus find

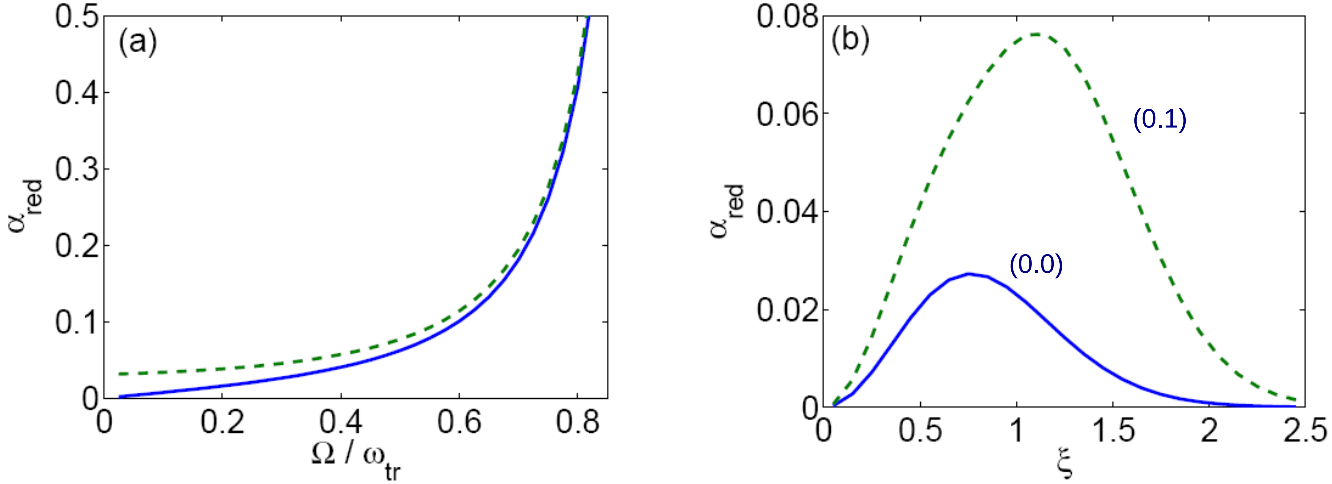
$$\alpha_a = \frac{1}{2\pi}\frac{\kappa^2 n_0}{L}\sum_{q\neq q_0}\frac{\varepsilon_q^0}{E_q^B}\frac{1}{(\hbar\omega_q)^2}\,\mathrm{e}^{-q^2\sigma^2/2}\sin(qa)$$

Workpackage 6:
Quantum simulation of CMP models in optical lattices (D12)

Parameters: BEC: ^{87}Rb with linear density of 5£10^{6}m^{-1}, circumference L=12μm
Lattice: 30 lattice sites with ^{23}Na atoms and coupling $\kappa/2aE_R$=0.035

Rotating 2D BEC

- Non-interacting rotating 2D BEC in a symmetric trap ω_{tr}



- Induced phase twist for the (0,0) BEC state

$$\alpha_{i,j}^{(0,0)} = \frac{\kappa^2 N_0}{2\pi^3 a_0^4} \sum_{\nu \neq \nu_0} \frac{1}{(\hbar\omega_\nu)^2} \frac{n!}{(n+l)!} e^{-\xi_i^2 - \xi_j^2} \times \xi_i^l \xi_j^l L_n^l(\xi_i^2) L_n^l(\xi_j^2) \sin(l(\varphi_i - \varphi_j))$$

Workpackage 6:
Quantum simulation of CMP models in optical lattices (D12)

- Where (ξ_i,ϕ_i) and (ξ_j,ϕ_j) are the initial and final particle polar coordinates

Lattice on a ring in a rotating BEC

- Description of the rotating BEC wave function

$$\phi_0 = \sqrt{n_0}\exp(\mathrm{i}q_0 x) \qquad q_0 = 2\pi j/L$$

where the integer j is chosen such that the BEC of chemical potential

$$\mu = \hbar^2 q_0^2/2m_b + gn_0 - \hbar v q_0 \qquad q_0 = m_b v/\hbar - \Delta q$$

$$\Delta q \in [-\pi/L, \pi/L]$$

forms the ground state

- The Bogoliubov excitation above this ground state are given by

$$u_q(x) = A_q \exp[\mathrm{i}(q + q_0)x]/\sqrt{L}$$

$$v_q(x) = B_q \exp[\mathrm{i}(q - q_0)x]/\sqrt{L} \qquad \hbar\omega_q = E_q^B - \hbar^2 q\Delta q/m_b$$

$$E_q^B = \sqrt{\varepsilon_q^0(\varepsilon_q^0 + 2gn_0)} \qquad \varepsilon_q^0 = \hbar^2 q^2/2m_b$$

Rotating 2D BEC

- Non-interacting rotating BEC eigenfunctions for symmetric trap ω_{tr}

$$\psi_{n,l}(r,\varphi) = \sqrt{n!/\pi(n+l)!}\,\mathrm{e}^{\mathrm{i}l\varphi}\mathrm{e}^{-\xi^2/2}\xi^l L_n^l(\xi^2)/a_0$$

$$E_{n,l}+\mu = (2n+l+1)\hbar\omega_{\mathrm{tr}}-\hbar\Omega l$$

$$\xi = r/a_0 \qquad a_0 = \sqrt{\hbar/m_b\omega_{\mathrm{tr}}}$$

- We consider $|\Omega|\cdot\omega_{tr}$ so that the ground state is ν=(n,l)=(0,0)

$$\alpha_{i,j}^{(0,0)} = \frac{\kappa^2 N_0}{2\pi^3 a_0^4}\sum_{\nu\neq\nu_0}\frac{1}{(\hbar\omega_\nu)^2}\frac{n!}{(n+l)!}\mathrm{e}^{-\xi_i^2-\xi_j^2} \times \xi_i^l\xi_j^l L_n^l(\xi_i^2)L_n^l(\xi_j^2)\sin(l(\varphi_i-\varphi_j))$$

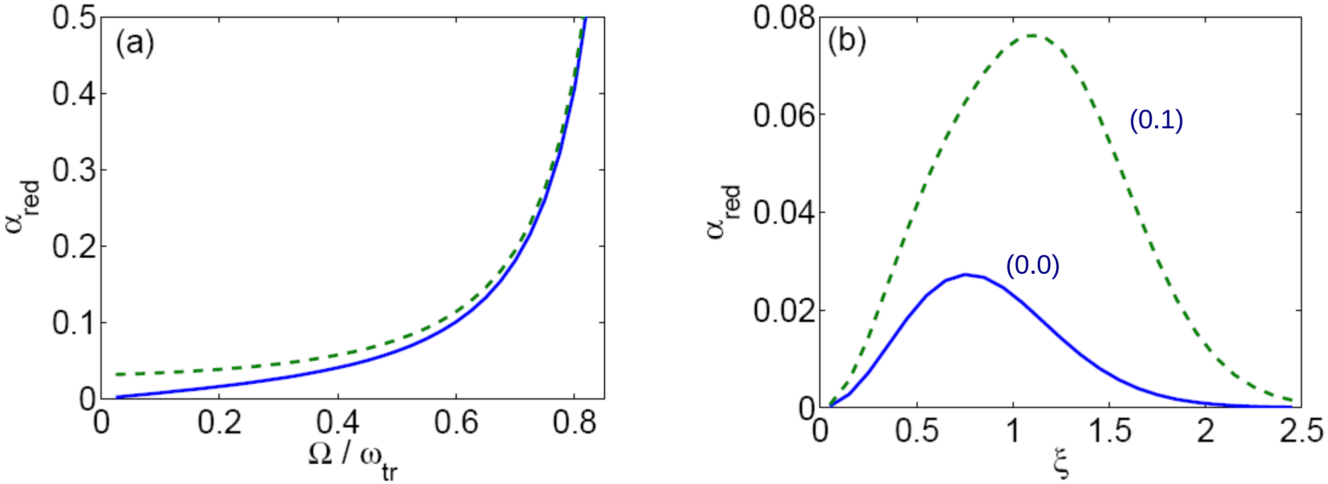
- Where (ξ_i,ϕ_i) and (ξ_j,ϕ_j) are the initial and final particle coordinates

BEC in a vortex state

- For the BEC in state (n,l)=(0,1) we find

$$\alpha_{i,j}^{(0,1)} = \frac{\kappa^2 N_0}{2\pi^3 a_0^4} \sum_{\nu \neq \nu_0} \frac{1}{(\hbar\omega_\nu)^2} \frac{n!}{(n+l)!} \mathrm{e}^{-\xi_i^2-\xi_j^2} (\xi_i \xi_j)^{l+1} \times L_n^l(\xi_i^2) L_n^l(\xi_j^2) \sin[(\varphi_i - \varphi_j)(l+1)] \, .$$

- The resulting phase twists are given by



$$\xi_i = \xi_j = 0.5$$

$$\varphi_i - \varphi_j = 0.1$$

$$\Omega = 0.25\omega_{\mathrm{tr}}$$

$$\alpha_{\mathrm{red}}^{(n,l)} = \alpha_{i,j}^{(n,l)} \left(\kappa^2 N_0 / 2\pi^3 a_0^4 (\hbar\omega_{\mathrm{tr}})^2\right)^{-1}$$

Conclusions

- Conclusions
  - Bell inequalities for high dimensional quantum systems
    - Optimal Bell inequalities
    - Applicable to squeezed states of optical lattices
    - Study connection between non-locality and entanglement

    } WP 4

  - Immersion of optical lattice in background BEC
    - Introduction of phonons
    - Self trapping and collapse of impurities
    - Quantum simulation of artificial magnetic fields

    } WP 6