Quantum simulators - WP 6

New theoretical strategies to implement QC - WP 7

Characterization of multi-particle entanglement - WP 4

Wolfgang Dür

Maarten Van den Nest

Akimasa Miyake

Otfried Gühne

Hans-J. Briegel

University of Innsbruck

Institute for Quantum Optics and Quantum Information (ÖAW)

(1) WP6 - Noisy quantum simulation & entanglement purification

D12,M6.1,M7.3

- **Simulation of many-body interactions using different methods**

WP7 - D14,M7.1

- **Effect of noise on quantum simulation protocols**
- **Multiparticle entanglement purification**

(2) WP7 - Measurement-based QC: universal resources

M7.2

- **The one-way model & graph states**
- **Universality: criteria and examples**
- **Classical simulation vs universality**
- **Graph states as ground states of local Hamiltonians**

(3) WP4 - Entanglement detection & computational methods

D9,M4.5

- **Methods for multiparticle entanglement detection**
- **Variational methods for ground-state approximation: Weighted graph states**

(1) Quantum simulation: many-body interactions

Fixed interaction Hamiltonian and (fast) local control

$$\prod_k (U_k e^{-i\delta t H} U_k^\dagger) \approx e^{-i\delta t (\sum_k U_k H U_k^\dagger)}$$

Trotter-Suzuki formula

Can generate new effective Hamiltonian corresponding to different system. In particular: any 2-body Hamiltonian for qubits

Many-body interactions:

$$H = \sigma_z \otimes \sigma_z \otimes \sigma_z$$

- simulation of higher dimensional systems (e.g. spin 1) using qubits

- simulation of fermionic systems

(via mapping to qubits with many-body interactions)

- fault tolerant adiabatic quantum computation

(a) Higher order terms in Trotter-Suzuki expansion:

$$e^{-i\delta t H_2} e^{-i\delta t H_1} e^{i\delta t H_2} e^{i\delta t H_1} \approx e^{i2\delta t^2 \times (-i/2)[H_1, H_2]}$$

(b) Teleportation based method:

use multiparticle entangled states (generated by 2-body interaction) as resource to generate many-body Hamiltonian by means of teleportation

(c) Graph state encoding:

apply *entangling* unitary operation U before and after evolution of system -> 1- and 2-body interaction terms are transformed into many body interaction terms

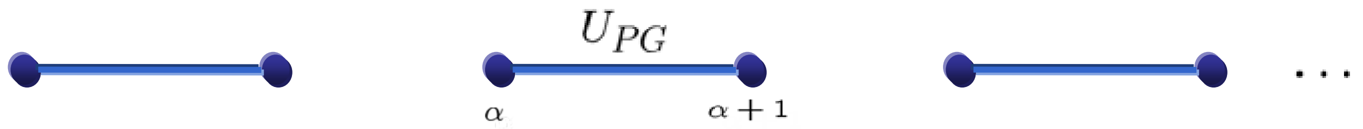
$$U e^{-itH} U^\dagger = e^{-itUHU^\dagger}$$

Graph state encoding

Choice of interaction pattern of U determines resulting interaction.
U can e.g. chosen to be some arrangement of controlled phase-gates,
easily implementable in optical lattice set-up.

Can generate:

- 3-body Hamiltonian with phase transition
-interacting d-level systems
- plaquette interactions on 2D lattice etc.



$$\sum_\alpha H = -(\sigma_z^{(\alpha-1)}\sigma_x^{(\alpha)} + \sigma_x^{(\alpha-1)}\sigma_z^{(\alpha)}) + B\sum_\alpha(\sigma_z^{(\alpha)}\sigma_x^{(\alpha+1)} + \sigma_x^{(\alpha)}\sigma_z^{(\alpha+1)})$$

$$UHU^\dagger = \sum_\alpha(-\sigma_z^{(\alpha-1)}\sigma_x^{(\alpha)}\sigma_z^{(\alpha+1)} + B\sigma_x^{(\alpha)})$$

Influence of noise in quantum simulation

we have investigated the influence of noise to generate many-body interactions using various noise models:

- timing errors in interactions
- phase noise and white noise in interactions

determine bounds & estimates, compare with numerical simulations

Higher order Trotter-Suzuki expansion:
noise in first order, desired Hamiltonian only in higher order -> noise increased by orders of magnitude

Graph state encoding:
fixed error independent of time (good for long t simulations)

Teleportation based methods:
Fixed error for each simulation step,
can use entanglement purification to significantly reduce noise !

Entanglement purification of graph states

Entanglement purification:

create highly entangled multiparticle states from several copies of noisy states via local operations and classical communication

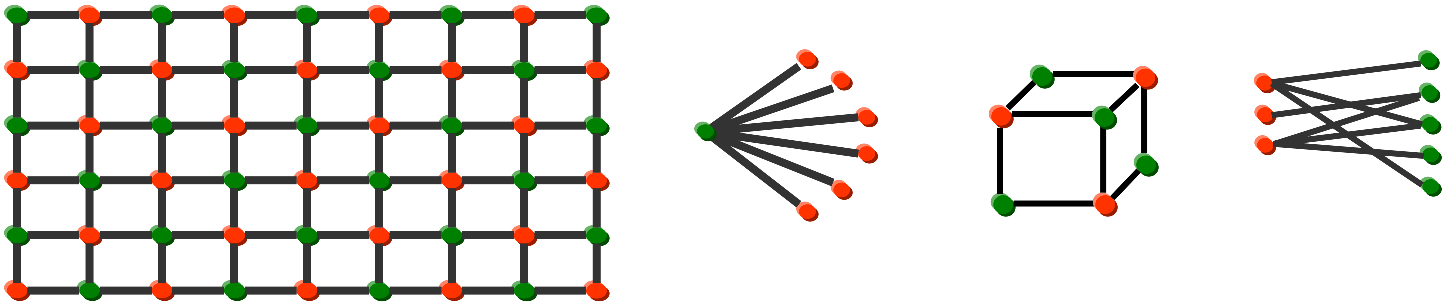
Results:

-**obtained *optimal* protocol to purify thermal graph states (purification regime, yield)**

-**established protocol to purify all graph states**

-established protocol for optical lattice set-up

Graph states



$$G = (V, E)$$

$$|\Psi\rangle \propto \prod_{k<l} e^{-i\pi/4 \sigma_z^{(k)} \otimes \sigma_z^{(l)}} |+\rangle^{\otimes N}$$

(2) Measurement based Quantum Computation: universal resources

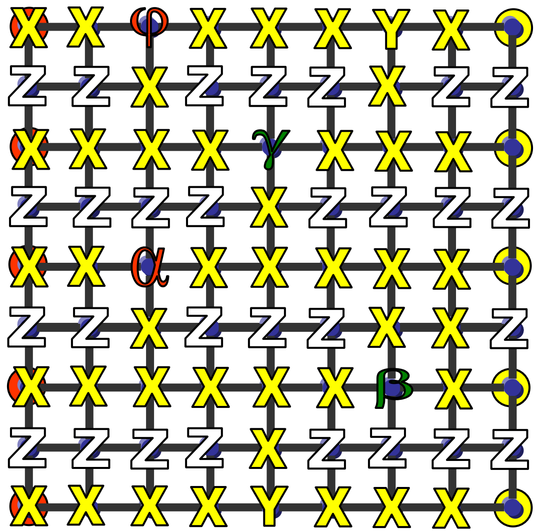
<u>One-way quantum computation</u>

(R. Raussendorf and H.-J. Briegel)

(i) preparation of 2D cluster state of size k x k

(ii) sequence of adoptive one-qubit measurements

(iii) remaining qubits are –up to LU- in desired state



$$|\psi\rangle = U|+\rangle^{\otimes N}$$

cluster state serves as *universal resource* for MQC

Universal resources

Which other states are universal for MQC ?
When is efficient classical simulation of MQC possible ?

Practical relevance:

- identify easily preparable resources
- identify resources that are stable under noise and decoherence
- identify simplest geometry (is 1D sufficient?)

Fundamental interest:

understand where the power of quantum computation comes from

Definition

A state ψ is called a univeral resource for MQC if *any* quantum state can be prepared by a sequence of local operations and classical communication (LOCC), i.e. $|\psi\rangle \geq_{LOCC} |\phi_{\text{out}}\rangle$.

Observations

A state Ψ is a universal resource for measurement based QC if and only if $|\psi\rangle \geq_{LOCC} |C_{k\times k}\rangle$ for every k.

A universal (set of) states Ψ needs to be maximally entangled with respect to all kinds of entanglement.
(entanglement can only decrease under LOCC)

-> obtain entangelment-based criteria for non-universality <-
If a certain state Ψ does not reach the maximal possible value for some entanglement measure, than it can not be universal.

Non-universal resources for MQC

The following states are not universal for MQC

1D cluster states
GHZ states
Graph states corresponding to tree graphs / with bounded tree width
W states
states with bounded block-wise entropic entanglement
etc.

at least one of the following measures is not maximal:

Schmidt-rank width
Geometric measure of entanglement
localizable entanglement
Schmidt measure

Universal resources for MQC

Other lattice structures

Graph states corresponding to Triangular lattice, Hexagonal lattice or Kagome lattice are universal resources for MQC

Cluster state with holes

(defects in lattice)

If probability for hole $p<0.41$, then one can show that the state is a universal resource fo MQC.

Encoded cluster states

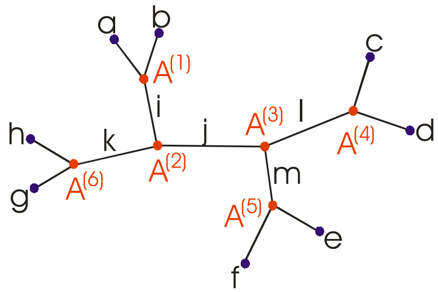
any encoded 2D cluster state (for any encoding) is a universal resource for MQC using only LOCC (single-qubit measurements!)

Classical simulation of MQC

Interpretation of Schmidt-rank width:

optimal description of a state in terms of a tree tensor network (TTN)

$$|\phi\rangle = \sum_{i_1,\ldots,i_n} A_{i_1\ldots i_n}|i_1\ldots i_n\rangle \quad i_\alpha = 1,2,\ldots,d_\alpha$$



Find optimal TTN for graph states

Classical simulation of MQC

MQC on all resource states with bounded Schmidt rank width
(1D cluster state, GHZ states, tree graphs etc.)
can be efficiently simulated classically

(coincides with criteria for non-universality!)

Graph states as ground states of local Hamiltonians

(Preparation of graph states – eg. 2D cluster state- by means of cooling)

- N-qubit Graph states cannot be exact, non-degenerate ground states of two-body Hamiltonians of a system of N qubits.

- If one allows for auxilliary particles, one can obtain any graph state as approximate ground state of a two-body Hamiltonian on the enlarged system (gadget construction).

(3) Entanglement detection & computational methods

Entanglement detection in many-body systems

-detection of GHZ and W states with linear overhead in system size

-lower bounds on generic entanglement measures from measured expectation values

-detection of entanglement using spin-squeezing inequalities (based on 1st and 2nd moments of collective angular momentum)

-application of entanglement detection methods to experiments (trapped ions + linear optics)

O. Gühne, C.-Y. Lu, W.-B. Gao, J.-W. Pan, *A toolbox for entanglement detection and fidelity estimation,arXiv/0706.2432*

T. Konrad, O. Gühne, J. Audretsch, H. J. Briegel, *Parameter estimation for mixed states from a single copy*, Phys. Rev. A 75, 062101 (2007)

O. Gühne, M. Reimpell, R. Werner, *Estimating entanglement measures in experiments*, Phys. Rev. Lett. 98, 110502 (2007)

J. Korbicz, O. Gühne, M. Lewenstein, H. Häffner, C. F. Roos, R. Blatt, *Generalized spin squeezing inequalities in N qubit systems: theory and experiment*, Phys. Rev. A 74, 052319 (2006)

G. Toth, C. Knapp, O. Gühne, H.J. Briegel, *Optimal spin squeezing inequalities detect bound entanglement in spin models*, quant-ph/0702219

C.-Y. Lu, X.-Q. Zhou, O. Gühne, W.-B. Gao, J. Zhang, Z.-S. Yuan, A. Goebel, T. Yang, J.-W. Pan, *Experimental entanglement of six photons in graph states*, Nature Physics 3, 91-95 (2007)

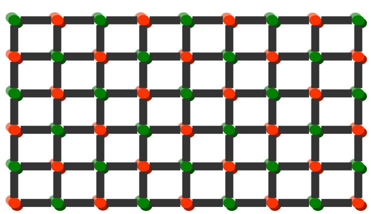
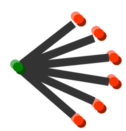
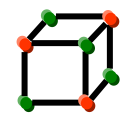
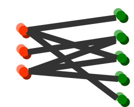
Ground state approximation for strongly interacting systems in arbitrary dimension

numerical optimization of

$$E_{\text{min}} = \min \langle \Psi_{\Gamma,d,U} | H | \Psi_{\Gamma,d,U} \rangle$$

--> Variational ansatz <-

$$|\Psi_{\Gamma,\chi,U}\rangle \propto \prod_k U_k \prod_{k,l} U_{kl}(\varphi_{kl}) \left(\sum_{i=1}^{m} |\chi_1^{(i)}\rangle |\chi_2^{(i)}\rangle \dots |\chi_N^{(i)}\rangle \right)$$

$$G = (V, E)$$

$$|\Psi\rangle \propto \prod_{k<l} e^{-i\varphi_{kl}\sigma_z^{(k)} \otimes \sigma_z^{(l)}} |+\rangle^{\otimes N}$$

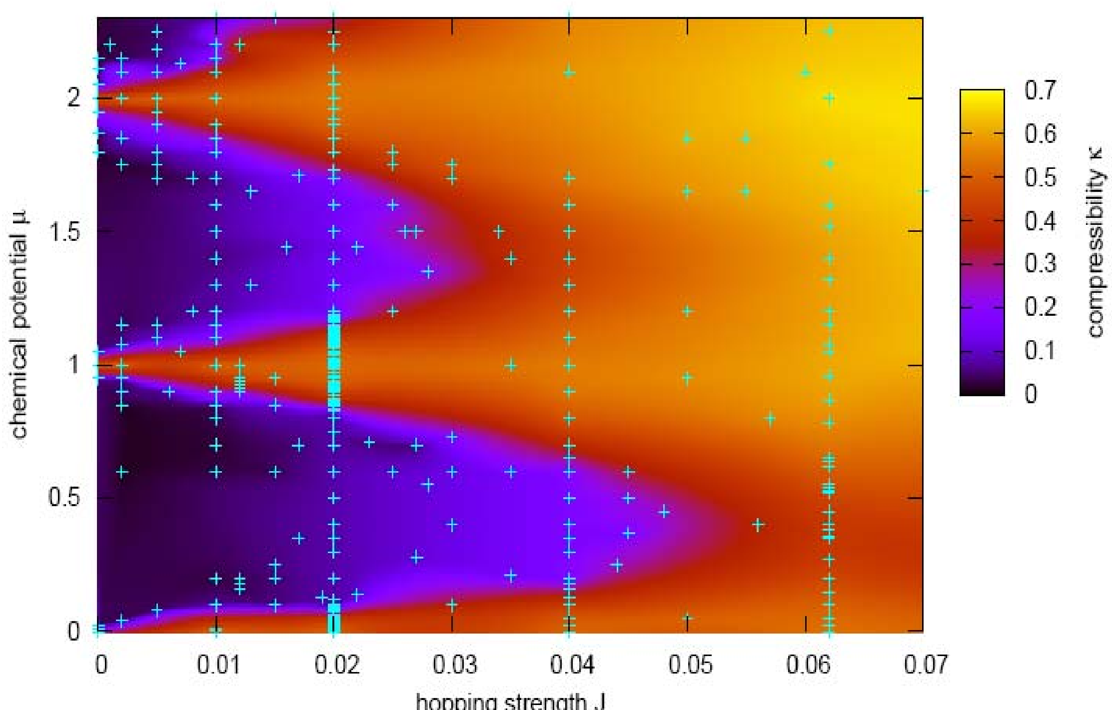
weighted graph states: edges of graph have weights:
interesting entanglement features (volume law etc.), long-range correlations,
highly adjustable to geometry (2D,3D)

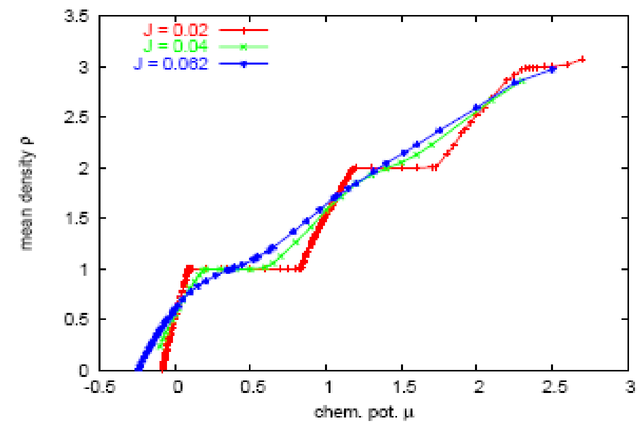
Example: Bose-Hubbard model

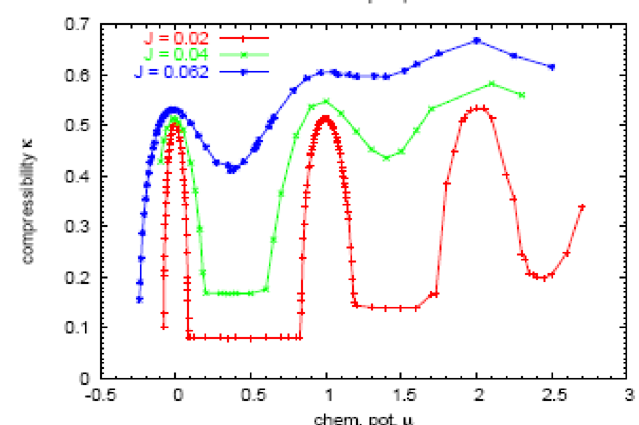
$$H = -J \sum_{(a,b)} \hat{b}_a^\dagger \hat{b}_b + U \sum_a \hat{n}_a(\hat{n}_a - 1)/2 + V \sum_{(a,b)} \hat{n}_a \hat{n}_b + \mu \sum_a \hat{n}_a$$

generalization of variational WGS method to higher dimensional systems possible

consider 2D Bose-Hubbard model, 4x4, with truncated occupation number







$$\kappa = \sqrt{\frac{1}{N} \sum_a \left(\langle \hat{n}_a^2 \rangle - \langle \hat{n}_a \rangle^2 \right)}$$

Combination with MPS, TTN and PEPS

$$|\Psi_{\mathrm{WGS}}\rangle \propto \prod_k U_k \prod_{k,l} U_{kl}(\varphi_{kl}) \left(\sum_{i=1}^{m} |\chi_1^{(i)}\rangle |\chi_2^{(i)}\rangle \dots |\chi_N^{(i)}\rangle \right)$$

$$|\Psi_{MPS}\rangle \propto \prod_k U_k \prod_{k,l} U_{kl}(\varphi_{kl}) |MPS\rangle$$

$$|\Psi_{PEPS}\rangle \propto \prod_k U_k \prod_{k,l} U_{kl}(\varphi_{kl}) |PEPS\rangle$$

can combine approach with Matrix product states (DMRG) and other tensor network structures (TTN,PEPS,MERA)

- time evolution
- mixed state algorithms

-> under development <-
potential applicability to predict experimental results (2D Bose- and Fermi- Hubbard models)

R. Hübener, C. Kruszinska, S. Anders, L. Hartmann, F. Verstraete, H. J. Briegel, J. Eisert, M. Plenio and W. Dür, in preparation

Publications

D12,M6.1,M7.3

W. Dür, M. Bremner and H.J. Briegel, *Quantum simulation of interacting high-dimensional systems: the influence of noise*, **E-print: arXiv:0706.0154**

WP6

Kay, J. Pachos, W. Dür and H.J. Briegel, *Optimal purification of thermal graph states,* **New Journal of Physics 8, 147 (2006).**

WP7 - D14,M7.1

C. Kruszynska, A. Miyake, H.J. Briegel and W. Dür, *Entanglement purification protocols for all graph states*, **Phys. Rev. A 74, 052316 (2006)**.

M. Van den Nest, A. Miyake, W. Dür, and H.-J. Briegel, *Universal resources for measurement-based quantum computation*, **Phys. Rev. Lett. 97, 150504 (2006);** *Fundamentals of universality in one-way quantum computation*, **New J. Phys. 9, 204 (2007).**

WP7

D. E. Browne, M. B. Elliott, S.T. Flammia, S.T. Merkel, A. Miyake, A.J. Short, *Phase transition of computational power in the resource states for one-way quantum computation*, **E-print: arXiv:0709.1729**.

M7.2

M. Van den Nest, W. Dür, G. Vidal and H.J. Briegel, *Classical simulation versus universality in measurement-based Quantum computation*, **Phys. Rev. A 75, 012337 (2007).**

M. Van den Nest, K. Luttmer, W. Dür and H.J. Briegel, *Graph states as ground states of many-body spin-1/2 Hamiltonians*, **E-print: quant-ph/0612186** (to appear in Phys. Rev. A)

O. Gühne, M. Reimpell, R. Werner, *Estimating entanglement measures in experiments*, **Phys. Rev. Lett. 98, 110502 (2007).**

WP4

D9,M4.5

S. Anders, M.B. Plenio, W. Dür, F. Verstraete and H.-J. Briegel, *Ground state approximation for strongly interacting systems in arbitrary dimension*, **Phys. Rev. Lett. 97, 107206 (2006).**

S. Anders, H. J. Briegel and W. Dür, *Variational method based on weighted graph states*, **New J. Phys. 9, 361 (2007).**

....